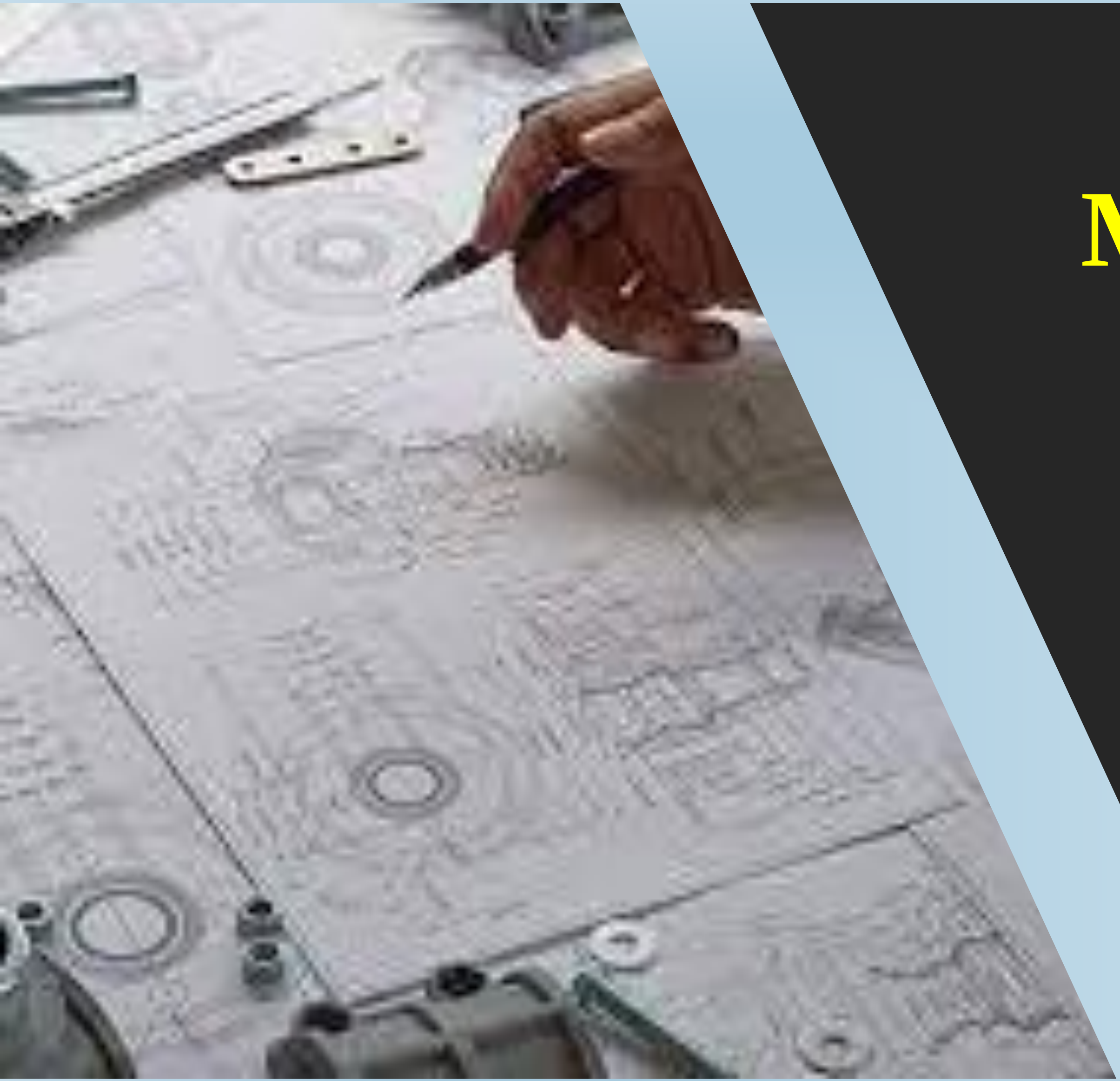
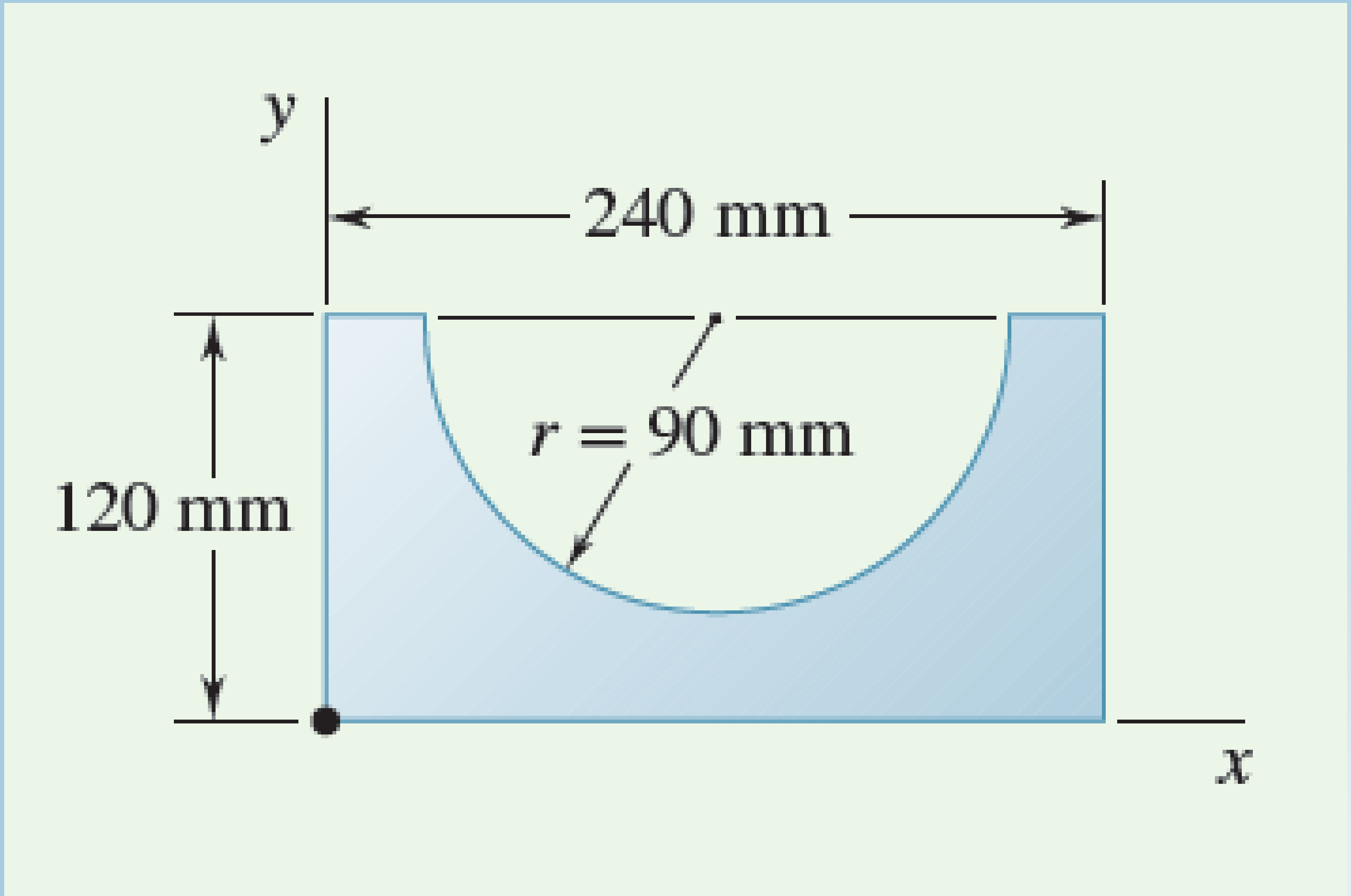


MECHANICS

Lecture No.2

د. محمد سعد

Determine the moment of inertia of the shaded area with respect to the *x* axis



MODELING and ANALYSIS:

Moment of Inertia of Rectangle. Referring to Fig. 9.12, you have

$$I_x = \tfrac{1}{3}bh^3 = \tfrac{1}{3}(240\text{ mm})(120\text{ mm})^3 = 138.2 \times 10^6\text{ mm}^4$$

Moment of Inertia of Half Circle. Refer to Fig. 5.8 and determine the location of the centroid C of the half circle with respect to diameter AA'. As shown in Fig. 2, you have

$$a = \frac{4r}{3\pi} = \frac{(4)(90\text{ mm})}{3\pi} = 38.2\text{ mm}$$

The distance b from the centroid C to the x axis is

$$b = 120\text{ mm} - a = 120\text{ mm} - 38.2\text{ mm} = 81.8\text{ mm}$$

Referring now to Fig. 9.12, compute the moment of inertia of the half circle with respect to diameter AA' and then compute the area of the half circle.

$$I_{AA'} = \tfrac{1}{8}\pi r^4 = \tfrac{1}{8}\pi(90\text{ mm})^4 = 25.76 \times 10^6\text{ mm}^4$$

$$A = \tfrac{1}{2}\pi r^2 = \tfrac{1}{2}\pi(90\text{ mm})^2 = 12.72 \times 10^3\text{ mm}^2$$

Next, using the parallel-axis theorem, obtain the value of $\bar{I}_{x'}$ as

$$I_{AA'} = \bar{I}_{x'} + Aa^2$$

$$25.76 \times 10^6\text{ mm}^4 = \bar{I}_{x'} + (12.72 \times 10^3\text{ mm}^2)(38.2\text{ mm})^2$$

$$\bar{I}_{x'} = 7.20 \times 10^6\text{ mm}^4$$

Again using the parallel-axis theorem, obtain the value of I_x as

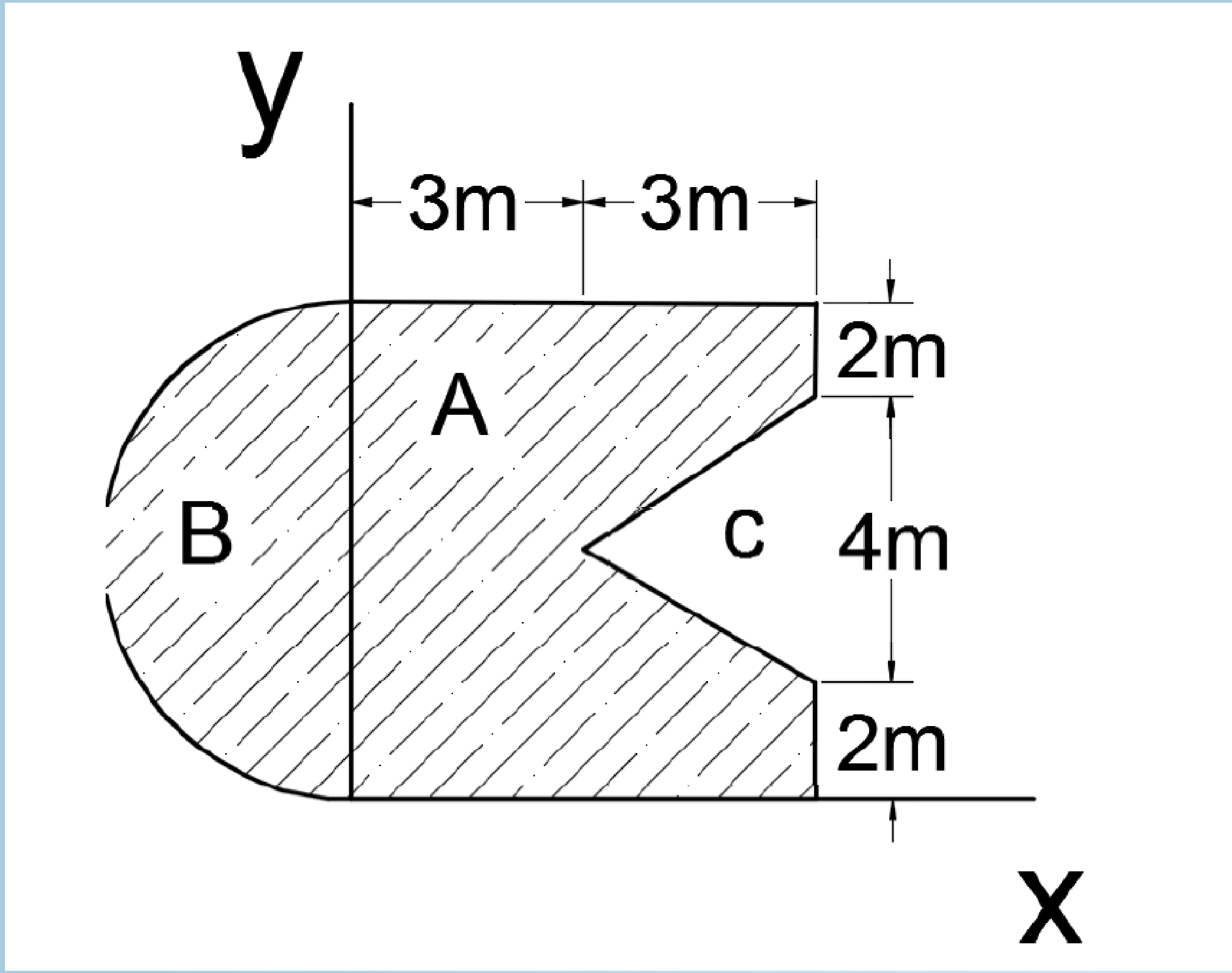
$$I_x = \bar{I}_{x'} + Ab^2 = 7.20 \times 10^6\text{ mm}^4 + (12.72 \times 10^3\text{ mm}^2)(81.8\text{ mm})^2$$
$$= 92.3 \times 10^6\text{ mm}^4$$

Moment of Inertia of Given Area. Subtracting the moment of inertia of the half circle from that of the rectangle, you obtain

$$I_x = 138.2 \times 10^6\text{ mm}^4 - 92.3 \times 10^6\text{ mm}^4$$

$$I_x = 45.9 \times 10^6\text{ mm}^4$$

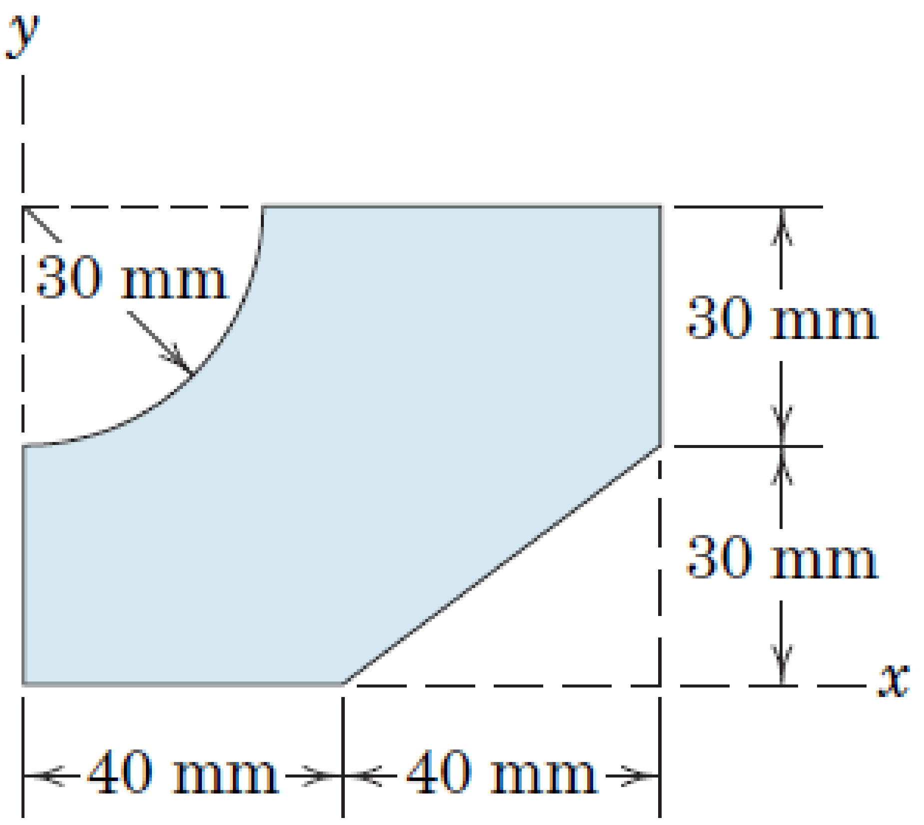
Determine the moment of inertia of the shaded area in the fig. below With y-axis



Solution:

$Iy_0(m^4)$	A (m^2)	$x^2(m^2)$	A. x^2	Iy (m^4)
$A = \frac{hb^3}{12} = 144$	48	9	432	576
$C = \frac{hb^3}{36} = -3$	-6	25	-150	-153
$B = 0.1098R^4 = 28$	25.12	2.88	72.5	100.5
Total				523.5

Determine the moments of inertia about the *x*- and *y*-axes for the shaded area. Make direct use of the expressions given in Table below for the centroidal moments of inertia of the constituent parts.



PART	A mm^2	d_x mm	d_y mm	Ad_x^2 mm^4	Ad_y^2 mm^4	$\bar{I}_x$ mm^4	$\bar{I}_y$ mm^4
1	$80(60)$	30	40	$4.32(10^6)$	$7.68(10^6)$	$\frac{1}{12}(80)(60)^3$	$\frac{1}{12}(60)(80)^3$
2	$-\frac{1}{4}\pi(30)^2$	$(60 - 12.73)$	12.73	$-1.579(10^6)$	$-0.1146(10^6)$	$-\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)30^4$	$-\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)30^4$
3	$-\frac{1}{2}(40)(30)$	$\frac{30}{3}$	$\left(80 - \frac{40}{3}\right)$	$-0.06(10^6)$	$-2.67(10^6)$	$-\frac{1}{36}\,40(30)^3$	$-\frac{1}{36}\,(30)(40)^3$
TOTALS	3490			$2.68(10^6)$	$4.90(10^6)$	$1.366(10^6)$	$2.46(10^6)$

$[I_x = \Sigma \bar{I}_x + \Sigma A d_x^2]$ $\quad I_x = 1.366(10^6) + 2.68(10^6) = 4.05(10^6)\ \text{mm}^4$ *Ans.*

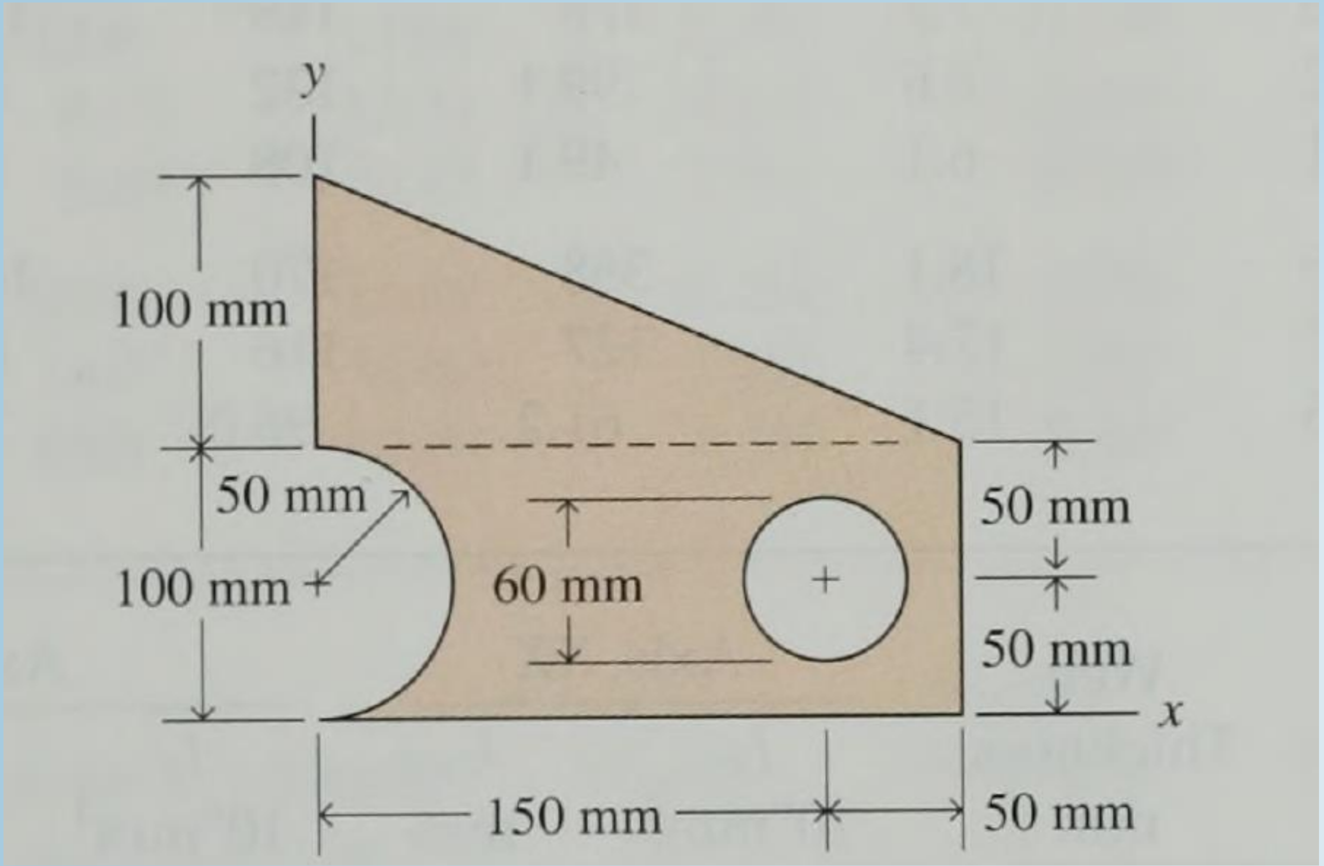
$[I_y = \Sigma \bar{I}_y + \Sigma A d_y^2]$ $\quad I_y = 2.46(10^6) + 4.90(10^6) = 7.36(10^6)\ \text{mm}^4$ *Ans.*

Determine the second moment of the shaded area shown in Fig. below with respect to

a. The x-axis.

b. The y-axis.

c. An axis through the origin O of the xy-coordinate system and normal to the plane of the area.



SOLUTION

As shown in Fig. 10-11*b*, the shaded area can be divided into a 100 × 200-mm rectangle (*A*) with a 60-mm diameter circle (*B*) and a 100-mm diameter half circle (*C*) removed and a 100 × 200-mm triangle (*D*). The second moments for these four areas, with respect to the *x*- and *y*-axes, can be obtained by using information from Table 10-1, as follows.

a. For the rectangle (shape *A*):

$$I_{x1} = \frac{bh^3}{3} = \frac{200(100^3)}{3} = 66.667(10^6)\ \text{mm}^4$$

For the circle (shape B):

$$I_{x2} = I_{xC} + \bar{y}^2 A = \frac{\pi R^4}{4} + \bar{y}^2(\pi R^2)$$

$$= \frac{\pi(30^4)}{4} + (50^2)(\pi)(30^2) = 7.705(10^6)\ \text{mm}^4$$

For the half circle (shape C):

$$I_{x3} = I_{xC} + \bar{y}^2 A = \frac{\pi R^4}{8} + \bar{y}^2\left(\frac{\pi R^2}{2}\right)$$

$$= \frac{\pi(50^4)}{8} + (50)^2\left[\frac{\pi(50)^2}{2}\right] = 12.272(10^6)\ \text{mm}^4$$

For the triangle (shape D):

$$I_{x4} = I_{xC} + \bar{y}^2 A$$

$$= \frac{bh^3}{36} + \bar{y}^2\left(\frac{bh}{2}\right)$$

$$= \frac{200(100^3)}{36} + \left(100 + \frac{100}{3}\right)^2\left[\frac{200(100)}{2}\right] = 183.333(10^6)\ \text{mm}^4$$

For the composite area:

$$I_x = I_{x1} - I_{x2} - I_{x3} + I_{x4}$$

$$= 66.667(10^6) - 7.705(10^6) - 12.272(10^6) + 183.333(10^6)$$

$$= 230.023(10^6) = 230(10^6)\ \text{mm}^4$$

Ans.

b. For the rectangle (shape A):

$$I_{y1} = \frac{b^3 h}{3} = \frac{200^3(100)}{3} = 266.667(10^6)\ \text{mm}^4$$

For the circle (shape B):

$$\begin{aligned} I_{y2} &= I_{yC} + \bar{x}^2 A \\ &= \frac{\pi R^4}{4} + \bar{x}^2(\pi R^2) \\ &= \frac{\pi(30^4)}{4} + (150^2)(\pi)(30^2) = 64.253(10^6)\ \text{mm}^4 \end{aligned}$$

For the half circle (shape C):

$$I_{y3} = \frac{\pi R^4}{8} = \frac{\pi(50^4)}{8} = 2.454(10^6)\ \text{mm}^4$$

For the triangle (shape D):

$$I_{y4} = \frac{bh^3}{12} = \frac{100(200^3)}{12} = 66.667(10^6)\ \text{mm}^4$$

For the composite area:

$$\begin{aligned} I_y &= I_{y1} - I_{y2} - I_{y3} + I_{y4} \\ &= 266.667(10^6) - 64.253(10^6) - 2.454(10^6) + 66.667(10^6) \\ &= 266.627(10^6) = 267(10^6)\ \text{mm}^4 \qquad \text{Ans.} \end{aligned}$$

c. For the composite area:

$$J_z = I_x + I_y$$
$$= 230.023(10^6) + 266.627(10^6)$$
$$= 496.650(10^6) = 497(10^6)\ \text{mm}^4 \qquad \text{Ans.}$$

Thank you for listening